Control of Distillation Columns with Sharp Temperature Profiles

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Temperature control problems are frequently encountered in distillation columns that, as a result of high purity products and an easy separation, have large temperature changes over a few trays. The describing function technique is used in this paper to analyze the stability of this type of column in which a nonlinear feedback control loop for column temperature control manipulates vapor boilup.

The cycling usually experienced in these systems is shown to be unavoidable in many cases because the process transfer function has such a high gain that it acts like an on-off switch. The process also displays nonlinear dynamics: system time constants vary with the magnitude of change in vapor boilup.

To overcome these difficulties an adaptive, nonlinear feedback controller is proposed that keeps the loop in a small-amplitude limit cycle by adaptively changing controller gain. The system is evaluated by digital simulation of a nonlinear model of the column.

Many types of distillation column control systems and problems have been discussed in the literature. One of the most popular topics has been the control of columns with very flat temperature profiles, that is, little temperature change per tray, which occurs in columns making difficult separations. Various schemes have been proposed to handle this situation: pressure compensation, differential vapor pressure cells, differential temperature control, and double-differential temperature control. See reference *4* for a recent summary.

On the other hand, distillation columns with very sharp temperature profiles (that is, with a very large temperature change over a few trays) have received very little attention in the literature. These columns are found in many chemical plants. A common example is a dehydration column for removal of water from a heavier organic compound. Product purities are high and the separation is an easy one (large relative volatilities). In some relatively short section of the column, tray compositions change from almost pure water to almost pure organic. Therefore the steady state temperature profile has a sharp break over a small number of trays.

Control problems are frequently experienced with these columns, usually in the form of apparently uncontrollable cycling in the feedback control loop maintaining a control tray temperature. Sometimes the cycling can be eliminated by going to very low feedback gains; but the resulting control loop is quite sluggish, giving large errors and long settling times.

One commonly used brute force technique for overcoming these problems is to overdesign the column (add more plates) and run on manual control. The temperature profile then moves up and down the column as disturbances enter the column or if there is any imbalance in the overall heat and material balances. Small, frequent changes must be made by the operator to keep the profile from going out one end of the column or the other. This technique is therefore not too desirable because it requires more operator attention and increases capital costs. Also if the column is pushed for capacity or separation, this system is unworkable.

The purpose of this paper is to explore this system to determine the basic cause of instability and to study ways to improve control effectiveness. Because the process is highly nonlinear, the describing function technique is used.

SYSTEM

The system studied as a specific numerical example was a 20-tray, equimolal overflow, constant pressure column in which water was being removed from a heavy organic. Two organics were considered: one boiling at 262 °F. and the other at 312 °F. under atmospheric pressure. Raoult's law was assumed. Saturated liquid feed and reflux and theoretical trays were assumed. Steady state values of variables and parameters are given in Table 1. Control tray temperature was maintained by manipulation of vapor boilup. The control tray for the 262 °F. organic was chosen as tray 7 and for the 312 °F. organic tray 5, following the usual procedure of selecting a control tray in the steep portion of the profile.

TABLE 1. PARAMETERS AND STEADY STATE VALUES

	Case 1	Case 2
Boiling point organic, °F.	312	262
Vapor boilup, moles/min.	75.1465	213.79
Reflux rate, moles/min.	25.1465	163.79
Control tray number	5	7
Control tray temperature, °F.	266.12	252.00

Number of trays	20	Distillate composition	0.999
Feed plate	10	Bottoms composition	0.001
Feed rate, moles/min.	100	Feed composition	0.500
Distillate rate, moles/min.	50	Tray holdup, moles	10
Bottoms rate, moles/min.	50	Reflux drum holdup, moles	100
		Reboiler holdup, moles	100

STEADY STATE ASPECTS

The first area explored was the steady state effects of changes in vapor boilup on the column profile. Figures 1 and 2 show profiles for the two systems with several values of vapor boilup (expressed as a percent change from the base case boilup rate). Very small changes in vapor boilup produce large steady state changes in control tray temperature.

This occurs because a small increase in vapor boilup moves the profile up the column a few trays. Since there is a large temperature change per tray in this section of the column, a small displacement produces a large temperature change as the tray composition changes to essentially pure organic (if the profile has moved up) or pure water (if the

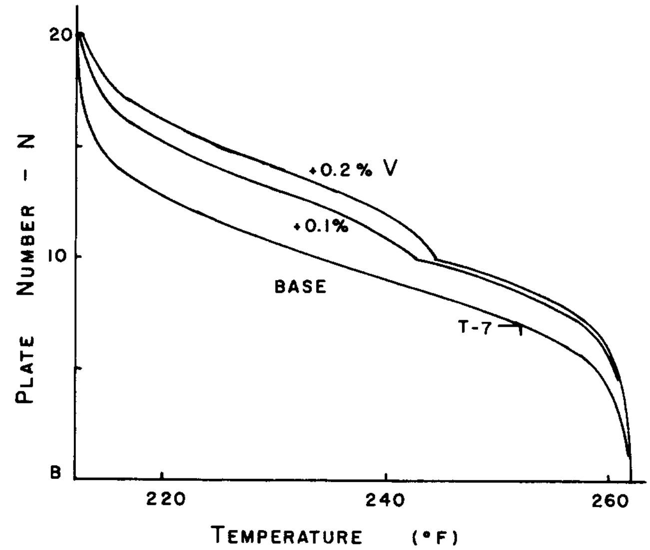

Fig. 1. Steady state temperature profiles (262°F.; organic).

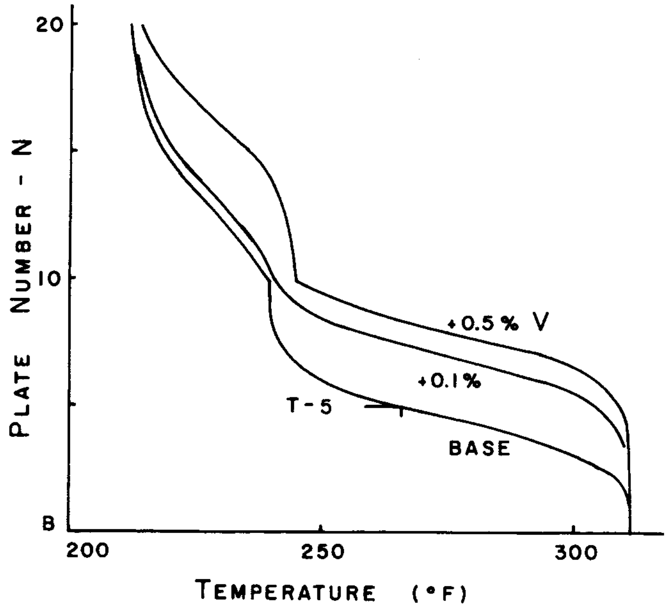

Fig. 2. Steady state temperature profiles (312°F.; organic).

profile has moved down). Any further change in vapor boilup in the same direction produces little change in temperature on the control tray because it is already almost pure organic or pure water.

Therefore the steady state gain for the transfer function between tray temperature and vapor boilup can be approximately represented by a linear gain with saturation. As shown in Figure 3 for the 312 °F. organic, the gain is equal to β for small steady state changes in vapor boilup V. For larger values of V, the process gain decreases because it is the ratio of the output to the input, and the input V is increasing while the output is constant. The gain will go to zero as V goes to infinity.

Thus the process steady state gain is distinctly nonlinear, and conventional linear stability analysis techniques cannot be realistically employed.

$$\begin{aligned} K &= \beta && \text{for } |V| \leq \Delta T/2\beta \\ K &= \Delta T/2|V| && \text{for } |V| > \Delta T/2\beta \end{aligned} \qquad (1)$$

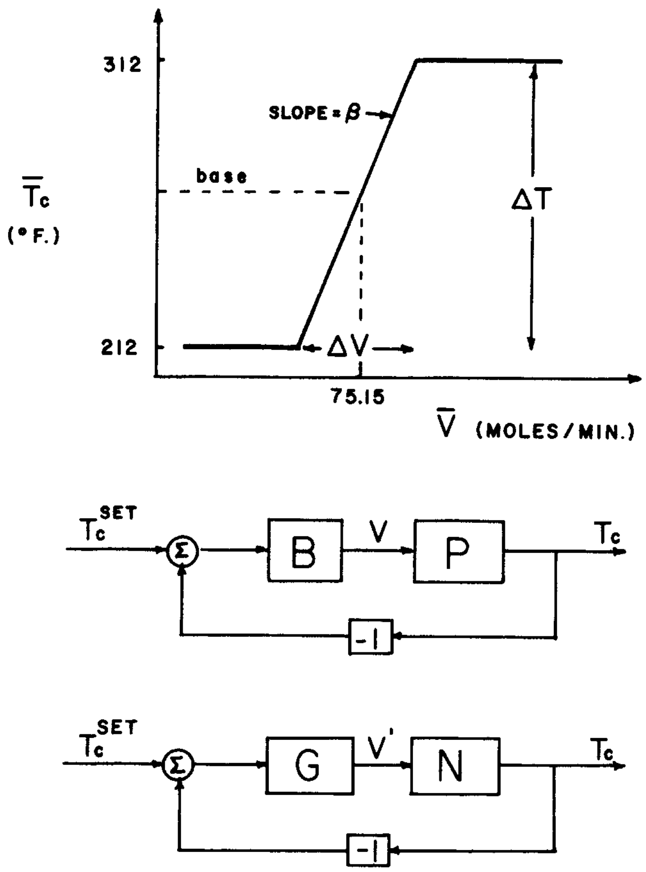

Fig. 3. Describing function nonlinear gain and block diagrams of original and reduced system.

DESCRIBING FUNCTION ANALYSIS

Describing function analysis has been widely used in electrical and mechanical systems to examine nonlinear effects, and is developed in the standard control theory texts (see for example reference *2*). Leder and Butt (*3*), in apparently the only chemical engineering application published to date, used describing functions to analyze tubular reactor dynamics.

The technique is approximate in that it considers only sinusoidal inputs and neglects higher harmonics. But it does permit the prediction of limit cycles and instability. The loop is broken up into a linear element $G_{(i\omega)}$ and a single nonlinear element $N_{(i\omega, |V'|)}$, see Figure 3. The technique has been extended to two nonlinear elements in reference *1*.

Plots are made of $(1/G)$ as a function of frequency ω and $(-N)$ as a function of frequency and the magnitude of the input to N, which in our case is the magnitude of V'. See Figures 5 and 6. The stability criterion is

$$\left|\frac{1}{G}\right| > |-N| \quad \text{when } \arg(1/G) = \arg(-N) \qquad (2)$$

To determine G and N for the distillation column under study (with the 312 °F. organic), the dynamic response of the control tray temperature to step changes in vapor boilup rates of different magnitudes was obtained by numerical integration of the rigorous nonlinear differential equations describing the column. A deadtime of 0.4 min. was assumed to exist in the temperature/vapor boilup loop, due to delays and lags in reboiler heat transfer, control valve, and sensor.

Results are shown in Figure 4. Response time was found to depend on the magnitude and, to a lesser extent, on the direction of the vapor boilup change. This would not occur, of course, if the system were linear. The re-

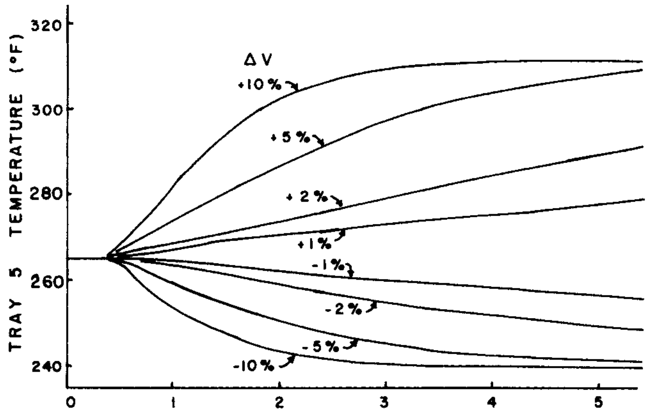

Fig. 4. Open-loop transient response for different magnitude vapor boilup step changes.

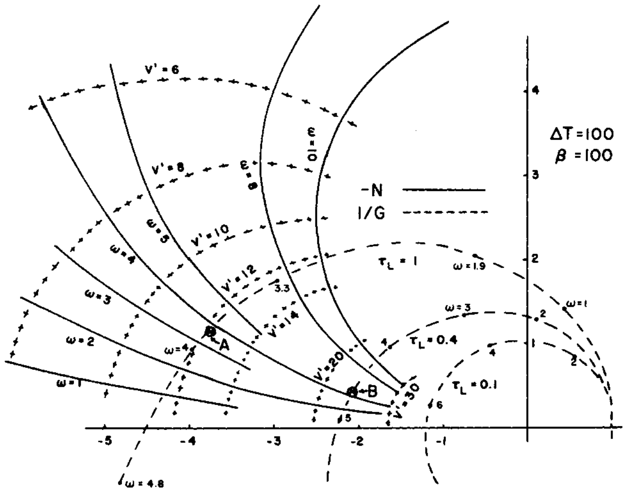

Fig. 5. Describing function plots.

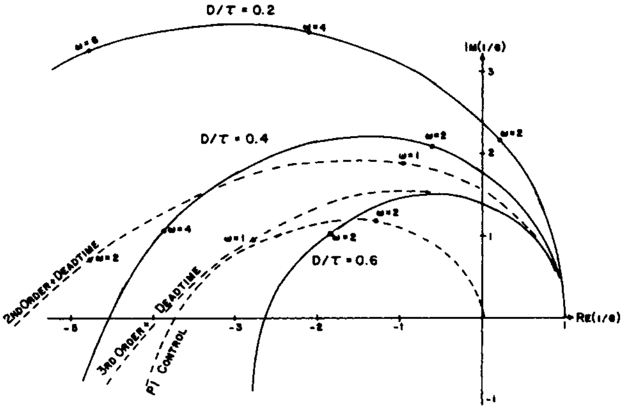

Fig. 6. 1/G plots.

sponse is nonlinear because the rate of change of the inventory of organic in the column depends upon the size of the vapor boilup change.

A nonlinear element must therefore be used that has a nonlinear steady state gain and nonlinear dynamics. The shapes of the curves in Figure 4 indicate that this nonlinear process (the transfer function P in the upper block diagram in Figure 3) can be approximated by linear components (deadtime D and first-order lag with a time constant τ_L) and nonlinear components (variable gain K and a first-order lag with a variable time constant τ_N). Gain K varies with the magnitude of V as given in Equation (1). Time constant τ_N should vary inversely with the magnitude of V, so a simple relationship was assumed:

$$\tau_{N(|V|)} = \frac{1}{|V|} \tag{3}$$

Therefore

$$P_{(s)} = \frac{\exp(-Ds)}{(\tau_L s + 1)} \frac{K}{(\tau_N s + 1)}$$

Splitting out the nonlinear elements of P and defining them as N

$$N_{(i\omega, |V|)} = \frac{K_{(|V|)}}{1 + i\omega\tau_{N(|V|)}} \tag{4}$$

Plots of $(-N)$ are given in Figure 5 for a $\beta = 100$ °F./(mole)(min.) and $\Delta T = 100$ °F.

The linear elements were lumped together to give

$$G_{(i\omega)} = \left[\frac{\exp(-iD\omega)}{1 + i\omega\tau_L}\right] \cdot [B_{(i\omega)}] \tag{5}$$

The system has now been reduced to the usual describing function form as shown in the lower block diagram in Figure 3. Figure 6 gives $(1/G)$ plots for several values of D/τ_L and for P and PI controllers with $K_c = 1$ and $\tau_I = 5$ min. The effects of increasing system order are also shown. The $(1/G)$ plots for several values of τ_L with proportional-only controllers and $D = 0.4$ are plotted in Figure 5 with the $(-N)$ curves. A family of curves must be used because N varies with both $|V|$ and frequency ω.

For $\tau_L = 1$, these describing function plots show an intersection of the $(1/G)$ and $(-N)$ curves at point A where $\omega = 3.8$ and $|V| = 12$. Since the system is right at the limit of stability at this point [from Equation (2)], we must now determine what the system will do at this limit, that is, will it be unstable or will it be stable?

Suppose we are at point A and a disturbance begins to move the system toward instability, that is, to increase the amplitude of oscillation $|V|$. Increasing $|V|$ means that we move down the $(-N)$ curve toward the origin and away from the $(1/G)$ curve. But stability criterion, Equation (2), tells us that the system is stable if $|1/G| > |-N|$. Therefore an increase in $|V|$ makes the system more stable, so the amplitude of oscillation will tend to decay, driving the system back to point A.

If a disturbance begins to move the system toward more stability, that is, reducing $|V|$, the opposite effect will occur. Now we move out the $(-N)$ curve away from the origin and into an unstable region where $|1/G| < |-N|$. So the amplitude of oscillation will grow, driving us back to point A.

Thus the describing function plots predict that a stable limit cycle will be set up. If $\tau_L = 0.4$, a stable limit cycle will occur at point B with a frequency $\omega = 4.6$ and amplitude $|V| = 25$.

The constant amplitude lines (the crosshatched lines) of $(-N)$ in Figure 5 intersect the negative real axis (when $\omega = 0$) at $-\beta$ for $|V| = \Delta T/2$. For very small values of β, the $(-N)$ curves may be entirely inside the $(1/G)$ curve; so the system would be stable with no limit cycle. However for large values of β, the $(-N)$ and $(1/G)$ curves must intersect, unless $(1/G)$ is very large (which corresponds to a very small feedback controller gain). In the limit as ΔT becomes a step function of V, β goes to infinity.

Therefore for many columns with sharp temperature profiles it may be physically impossible to not get limit cycle behavior with a closed-loop feedback system.

ADAPTIVE FEEDBACK CONTROL

Faced with the rather discouraging conclusion that cycling may be unavoidable in many distillation columns with sharp temperature profiles, clearly we will have to employ a control system that can accommodate cycling. A control scheme that comes immediately to mind is the adaptive high-gain controller used in flight systems (*5*). The basic strategy is to keep the loop in a small-amplitude limit cycle by adaptively changing the feedback controller gain. Tight control can thus be maintained despite changes in process parameters due to set point and load disturbances. This prevents sluggish response at some operating levels and instability at others that may be experienced with a fixed-gain controller.

When the amplitude of oscillation begins to increase, controller gain is decreased; when the amplitude decreases, gain is increased. Several techniques for making this adjustment were tested on the nonlinear column model. The most effective was found to be one that averaged two previous values of amplitude (detected by a peak picker) and adjusted the gain by the algorithm:

$$K_c = K_{co}(A_o/A)^{0.5} \tag{6}$$

where

$$A = (A_n + A_{n-1})/2$$

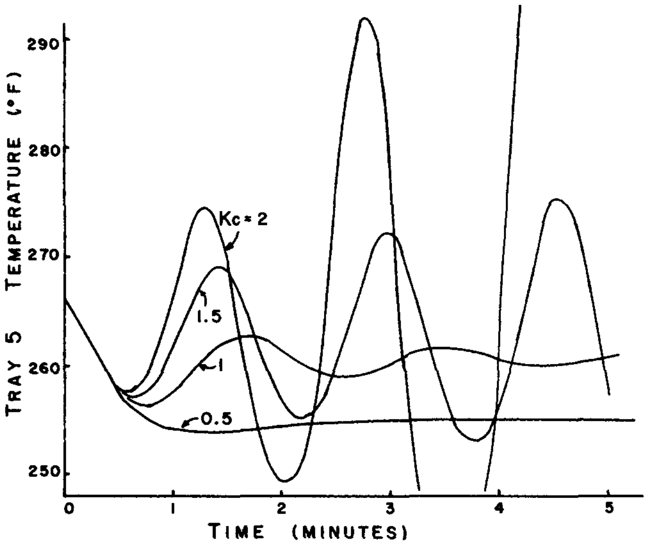

Fig. 7. Closed-loop response with proportional-only controller.

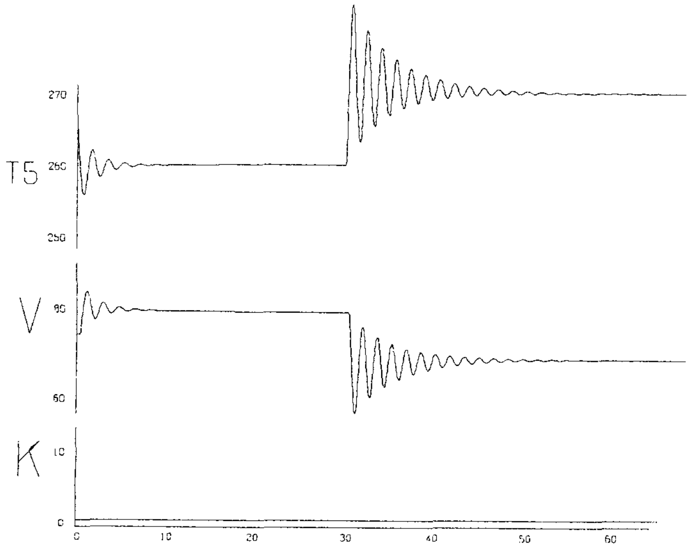

Fig. 8. Closed-loop response for feed rate disturbances with proportional-only control.

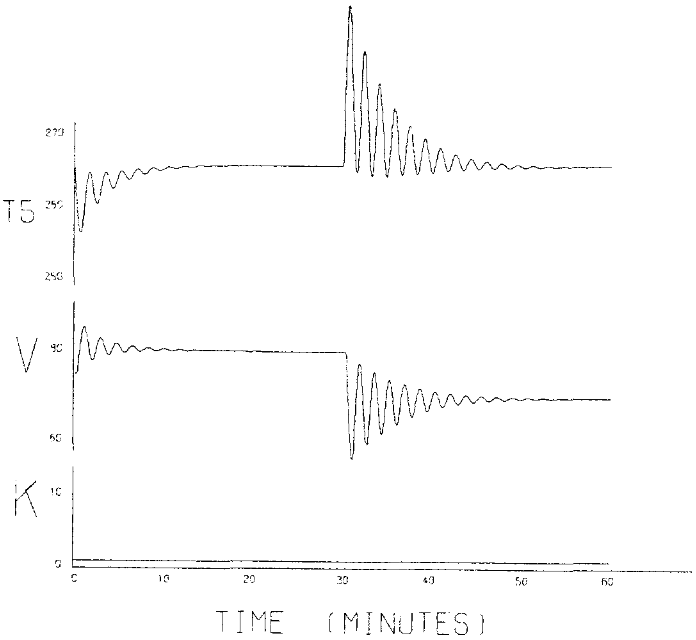

Fig. 9. Closed-loop response for feed rate disturbances with proportional-integral control.

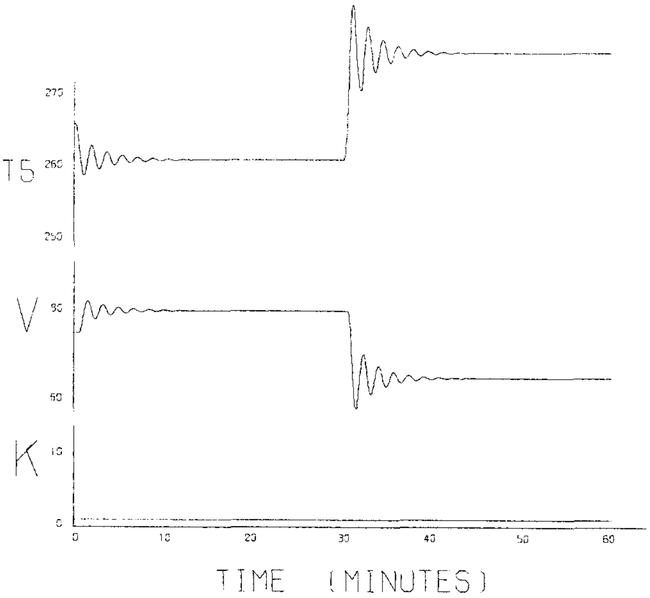

Fig. 10. Closed-loop response for feed composition disturbances with proportional-only control.

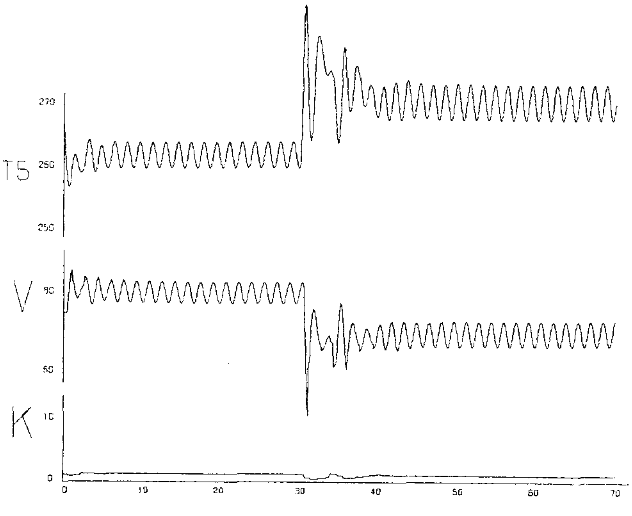

Fig. 11. Closed-loop response with proportional adaptive control.

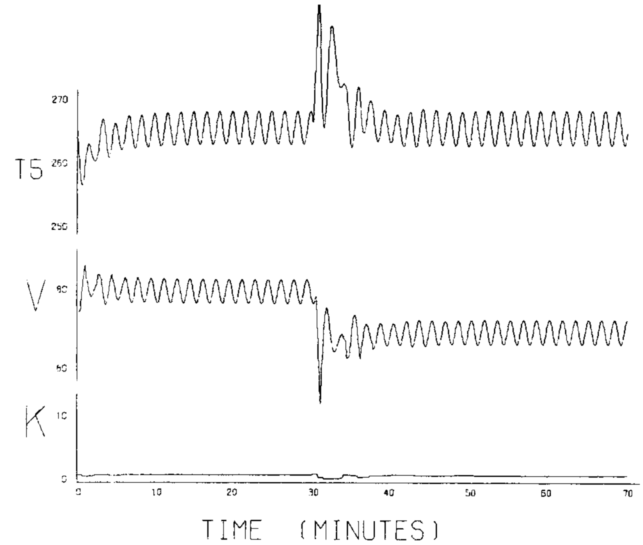

Fig. 12. Closed-loop response with proportional-integral adaptive control.

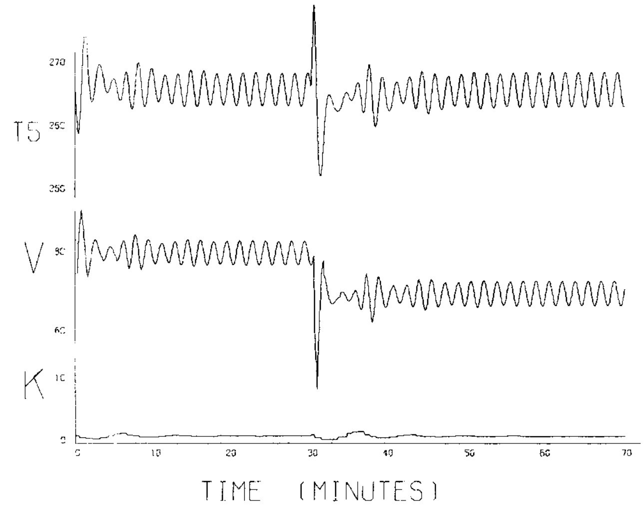

Fig. 13. Closed-loop response with proportional-integral feedback and steady state gain feedforward control.

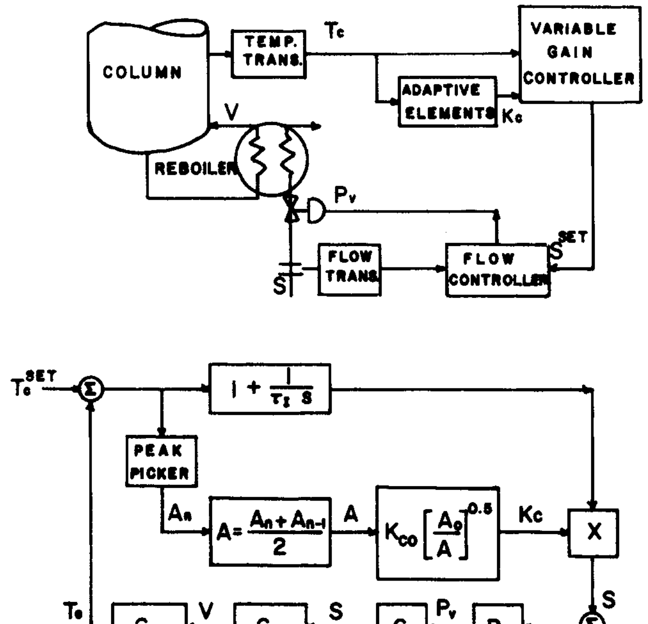

Fig. 14. Adaptive controller block diagram.

The system is sketched in Figure 14.

With conventional fixed-gain proportional-only control, feedback gain is limited to about 1, as shown in Figure 7, for a step disturbance in feed rate from 100 to 110 moles/min. These curves were obtained from a digital simulation of the nonlinear model of the column, using simple Euler numerical integration with at step size of 0.01 min.

Figures 8, 9, and 10 show the effect of changing load on system damping. In Figures 8 and 9 feed rate is changed from 100 to 110 moles/min. at $t = 0$ and from 110 to 90 moles/min. at $t = 30$ minutes. Figure 8 is with proportional control with a gain of 1. Figure 9 is with proportional-integral control with a gain of 1 and reset time of 5 min. Figure 10 is proportional only for a feed composition disturbance from 0.50 to 0.55 mole fraction at $t = 0$ and from 0.55 to 0.40 at $t = 30$ min. Damping decreases (that is, the system becomes more oscillatory) as feed rate decreases and as feed composition decreases.

The same feed rate disturbances were introduced into the system with the adaptive controller in Figures 11, 12, and 13. Figure 11 is with proportional only control; Figures 12 and 13 are with *PI* control, with Figure 13 including a steady state gain feedforward controller. The adaptive controller effectively changes the feedback gain K_c to keep the system oscillating about 5 °F. Table 2 gives the K_c values required for different feed rates.

TABLE 2. CRITICAL FEEDBACK GAINS

Feed rate, moles/min.	Controller gain
110	1.17
90	1.14
70	1.09
60	1.06

CONCLUSIONS

1. Cycling may be unavoidable in some distillation columns with very sharp temperature profiles.
2. Describing function techniques can be used to analyze these systems for stability and limit cycle formation.
3. An adaptive controller is effective in these systems in preventing both sluggish response and instability at different operating levels.

NOTATION

A = average amplitude of oscillation in control tray temperature, °F.
A_O = desired average amplitude of oscillation in control tray temperature, °F.
A_n = present (latest) value of amplitude of oscillation in control tray temperature, °F.
F = feed rate, mole/min.
B = feedback controller transfer function
D = deadtime in temperature/vapor boilup loop, min.
G = linear portion of describing function loop
$i = \sqrt{-1}$
K_c = feedback controller gain
K = gain of nonlinear part of the describing function loop
N = nonlinear portion of describing function loop
R = reflux rate, moles/min.
T_c, T_s = control tray temperature, °F.
T_c^{set} = control tray temperature set point, °F.
t = time, min.
V = total or perturbation in vapor boilup rate, moles/min.
V' = input to nonlinear portion N of describing function loop

$|V|$ = absolute value (magnitude) of oscillation in vapor boilup
X_B = bottoms composition, mole fraction water
X_D = distillate composition, mole fraction water
X_F = feed composition, mole fraction water
τ_N = nonlinear time constant, min.
τ_L = linear time constant, min.
β = slope of T_c versus V line between limits on T_c of $\pm\Delta T/2$, °F./(moles)(min.)
ω = frequency, rad./min.
$-$ = total steady state values of variables

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Mixing and Product Distribution for a Liquid-Phase, Second-Order, Competitive-Consecutive Reaction

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Mixing effects for the homogeneous, liquid-phase, second-order, competitive-consecutive reaction of iodine (B) with L-tyrosine (A) to form 3-iodo-L-tyrosine (R) and 3,5-diiodo-L-tyrosine (S) were determined for the following conditions: vessel volume, 5 and 36 liters (baffled and unbaffled); turbine diameter (2 to 6 in.) and speed (95 to 1,600 r.p.m.); feed inlet locations (3), addition rate (0.25 to 18 min.), and distribution; temperature (11° to 43°C.); initial A concentration (0.1 to 0.4 g.-mole/liter); and kinematic viscosity (0.765 to 6.35 centistokes). A was initially charged to the reactor and an equimolar quantity of feed B was added over a time period. Yields of R are less than that expected for perfect mixing owing to local regions of excess B concentration that exist for time periods during which R over-reacts to S.

Agitation power for a given yield is less in unbaffled vessels without an air-liquid interface than for baffled vessels. The local fluctuating velocity u' where feed is introduced correlates the mixing variables and predicts mixing requirements for maintaining yields of R on scale-up. Regions of excess B concentration are related to a concept of partial segregation. The extent of reaction occurring under this condition is correlated by the dimensionless group $(k_1 b \tau)(a_0/b)$, where τ is a microtime scale of mixing related to u' and the characteristic length of a microscale eddy. The magnitude of this group provides a criterion for predicting the importance of mixing effects on other reaction systems.

INTRODUCTION

The method of bringing together reactants that are to undergo reaction can have an influence on the course of the reaction for certain types of systems. If the reaction can result in only one product, the method can only influence the reaction rate. If more than one product is possible, contacting can influence the product distribution as well. These considerations apply to both homogeneous and heterogeneous reaction systems. This study is limited to homogeneous liquid-phase systems. The reactions studied form a system of the type

$$A + B \xrightarrow{k_1} R \tag{1}$$

$$B + R \xrightarrow{k_2} S \tag{2}$$

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